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Determining the Optimized Initial Angle and Velocity for Free Kicks in Soccer

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Abstract: **A direct Free Kick in soccer exemplifies the physics of projectile motion. This study investigated the effect of projection angle and initial velocity in achieving soccer free kick goal success rates. At a fundamental level, the initial angle, θ, and the initial velocity, V_0, with which the ball is kicked, determines the trajectory of the soccer ball and the time it takes to reach the goal. Using equations of projectile motion, as well as optimizing a modeled equation for reaching the optimal location within optimal time, various initial values of optimized θ and V_0 have been obtained for specific Free Kick distances with the appropriate constraints.**

Keywords: **free-kick, football, goal, minimization, optimization, projectile, soccer.**

I. INTRODUCTION

Soccer, or more widely termed as Football, has long been a cherished athletic activity in many countries across the world. First established in 1863 as a recreational pursuit, the sport has since developed into a global competitive phenomenon, with 209 countries fighting for the 32 coveted spots of the World Cup Finale every four years.[3] As the competition grows, players strive to perfect their game, utilizing advanced technology to improve their fitness, speed and technical skills. While training helps in executing offensive and defensive "plays" to some extent, these "plays" can never be exactly the same as practised in the training due to many variables in a real match scenario. That being said, one of the few areas of matches that can be exactly replicated is the Free Kick. A direct Free Kick is awarded when a player commits a deliberate foul outside of his or her own penalty area, and the ensuing kick is taken from where the foul was committed.[4] For this project, only direct free kicks are considered, wherein the kicker can score directly off the Free Kick.

II. PROJECTILE MOTION

A projectile is an object upon which the only force acting is gravity. Gravity acts to influence the vertical motion of the projectile, thus causing a vertical acceleration. The horizontal motion of the projectile is the result of the tendency of any object in motion to remain in motion at constant velocity. Projectile motion is considered when a projectile is thrown near the earth's surface, and moves along a curved path under the action of gravity only. Ideally, a football can be considered as a projectile and its path during a kick can be traced using the equations of projectile motion, which are as follows:

$$y = V_o t \sin(\theta) - \frac{1}{2} g\, t^2 \qquad \text{.......(1)}$$

$$x = V_o t\, cos(\theta) \qquad \text{.......(2)}$$

Where θ and V_0 are the initial angle and initial velocity with which the ball is kicked. Gravity is represented by g. Using both the aforementioned equations and substituting for time 't', we get the following equation:

$$y = tan(\theta)x - \frac{g}{(2\, V_o{}^2\, cos^2\theta)}\, x^2 \qquad \text{.......(3)}$$

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Here '**y**' is the vertical position of the ball and '**x**' is the horizontal position of the ball during its time of flight. The path of a projectile can generally be represented by the following graph, where V_x and V_y are the horizontal and vertical velocity of the ball and V_o and θ_o are the initial velocity and angle of the projectile and g is the gravity.

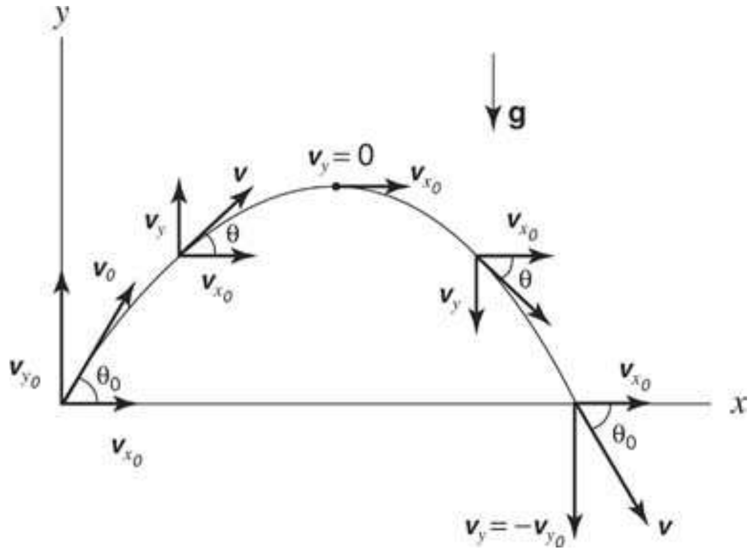

Fig.1. Projectile motion dynamics [2]

III. PARAMETERS AND CONSTRAINTS

In the case of a soccer Free Kick, the details of the situation can be represented by the following graph, which indicates the relevant distances and heights present. In soccer, the standard height of a goal post in an 11-a-side match is 2.44 metres. Also, the defender wall must stand a distance of 9.1 metres away from the ball during a Free Kick. The average height of a defender is 1.9 metres and that has been taken as the height of the defender wall. The specific distance that is considered for this project is 20 metres.

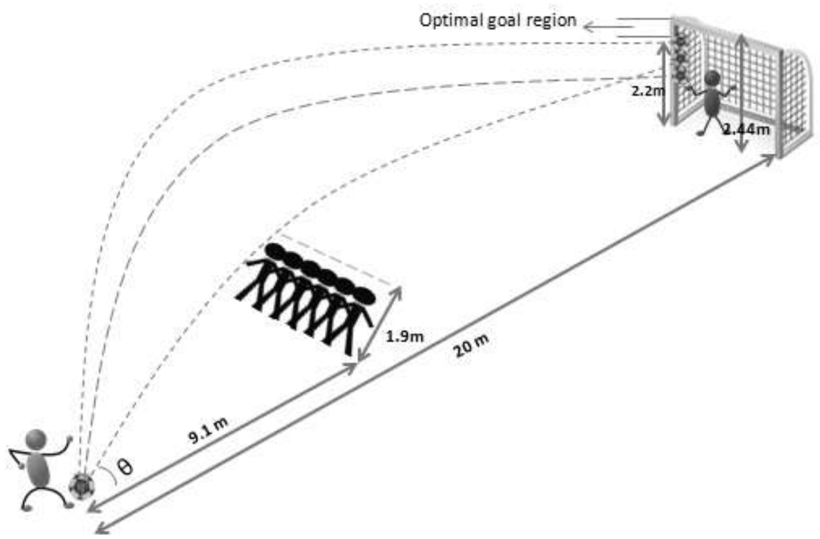

Fig. 2. Soccer Free Kick - projectile motion with constraints

Using the equations of projectile motion, data was obtained to mark the path of the football. Appropriate range constraints were applied to both the initial angle θ and the initial velocity V_{0}:

$$15° \leq \theta \leq 70°$$

$$15 m/s \leq V_0 \leq 40\ m/s$$

Also, a constraint has to be applied to filter out paths that are not able to clear the initial wall of 1.9m by the non-linear constraint derived from equation (3) above:

$$9.1 \times tan(\theta) - \left(\frac{g}{2V_0{}^2 cos^2\theta}\right) \times 9.1 \geq 1.9$$

(Here 9.1 is the horizontal distance(x) and 1.9m is the height of the wall) Using the above ranges for Initial angle (15 to 70 degrees) and initial Velocity (15 m/s to 40 m/s), the position (Y) of the ball can be determine when it reaches the goal (i.e. at X=20 meters).

IV. DATA ANALYSIS

The following table shows a sample of a larger table of data obtained for various angles (θ) and initial Velocity (V_o) calculating the corresponding position of the ball when it reaches the goal, while clearing the wall height.

Table 1 - Data calculations for free-kick projectile motion

Velocity	Angle	Error	Ball height at the Wall	Ball height at the goal
15	31	2.03890152	3.013319653	0.16109848
15	32	1.815079387	3.178727741	0.384920613
15	33	1.596698071	3.34563554	0.603301929
15	34	1.384163037	3.514123637	0.815836963
16	25	2.19489352	2.31371	0.00510648
16	26	1.922890544	2.476278358	0.277109456
16	27	1.653427416	2.640145656	0.546572584
16	28	1.386597471	2.805407585	0.813402529
17	22	2.008558164	2.043401313	0.191441836
17	23	1.714484937	2.205696607	0.485515063
17	24	1.421820831	2.369214207	0.778179169
17	25	1.130552549	2.534055189	1.069447451
17	26	0.84067983	2.700323031	1.35932017
18	21	1.463488222	2.056250033	0.736511778
18	22	1.156342442	2.219831273	1.043657558
18	23	0.849857331	2.384696137	1.350142669
18	24	0.543969569	2.550951365	1.656030431
18	25	0.238624463	2.718706601	1.961375537
19	20	1.069209413	2.039212298	1.130790587
19	21	0.752107474	2.203523632	1.447892526
19	22	0.435112311	2.369143941	1.764887689
19	23	0.118123023	2.536183432	2.081876977
20	19	0.79439911	1.998687322	1.40560089
20	20	0.469719539	2.163321689	1.730280461
20	21	0.144742127	2.329263443	2.055257873
20	22	0.18066228	2.496624676	2.38066228
20	23	0.506619863	2.665520828	2.706619863
……	….	…..	…..	……

We are defining ‘Error’ as the vertical distance between the football where it reaches the goal and 2.2m (the optimum height at the goal) at the horizontal distance of 20m (as the free kick is taken from a distance of 20m from the goal).

2.2 m is taken as the optimal location for the ball to enter the goal, allowing for the margin of goal height - ball diameter. If the centre of the ball passes through 2.2m, the ball should enter the goal near the top-most possible height, farthest away from the reach of the goalkeeper.

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Following graph represents the 'Error' variation with different values of initial velocity and angles calculated using projectile equations:

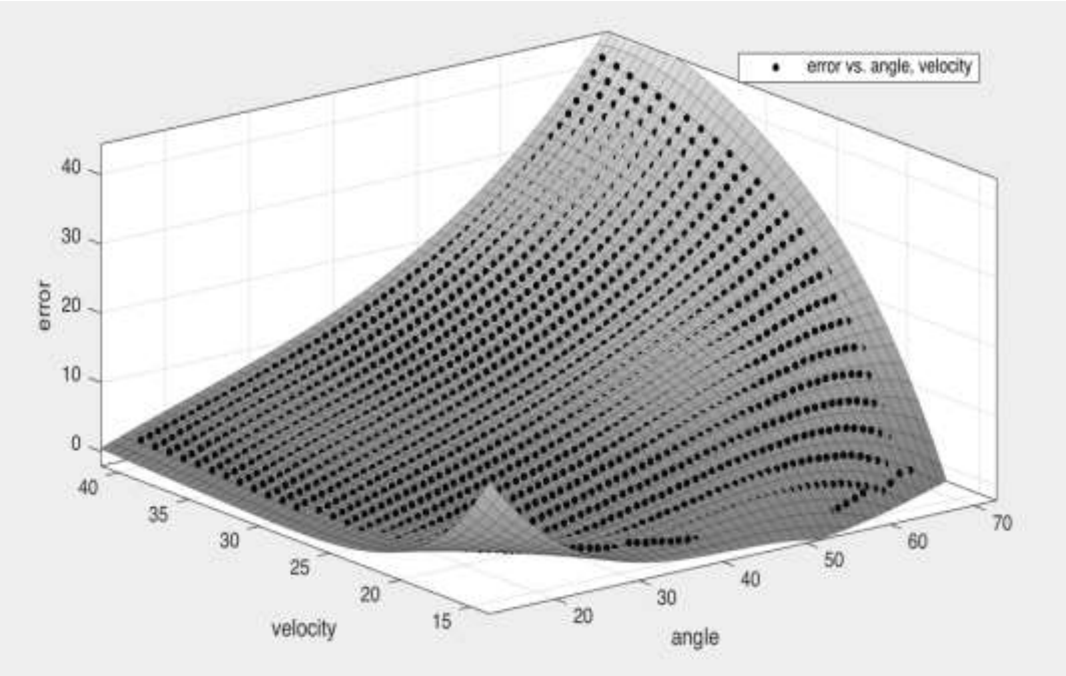

Fig. 3. Error Vs Velocity and Angle plot

V. ERROR OPTIMIZATION

We know that $y = tan(\theta)\,x - \frac{g}{(2\,V_0{}^2\,cos^2\theta)}\,x^2$

So, an Error equation can also be obtained by taking the absolute value of the vertical difference between the football position at the goal and 2.2m (the optimum position) at the horizontal distance of 20m, as stated earlier.

Mathematically the error constraint can be defined as:

Error $= |(tan(\theta) \times 20 - \left(\frac{g}{2V_0{}^2 cos^2\theta}\right) \times 20^2 - 2.2)|$

Manual Optimization attempt:

An attempt was made to optimise the error equation manually using Lagrange multipliers[1], but the result was inconsistent.

$$f = \left(20\tan(\theta) - \left(\frac{g}{2V_0{}^2 cos^2\theta}\right) \times 20^2 - 2.2\right)$$

constraint non-linear inequality restricted by the wall height:

$$g = 9.1\tan(\theta) - \left(\frac{g}{2V_0{}^2 cos^2\theta}\right) * 9.1 \;\geq\; 1.9$$

first take: $h = f + \lambda(g - 1.9)$

$$\frac{\partial h}{\partial V} = 0 \rightarrow \;2000\,sec^2\theta\,\times \frac{2}{V^3} + \lambda\left[\frac{405}{V^3} \times 2sec^2\theta\right]$$

$$2000 + 405\,\lambda = 0$$

$$=> \;\lambda = -\frac{2000}{405}$$

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$$\frac{\partial h}{\partial \theta} = 0 \rightarrow 20sec^2\theta \; - \frac{2000}{V^2} \times \; 2\,sec\theta\; sec\theta\; tan\theta \; + \lambda \left[9\; sec^2\theta - \frac{405}{V^2} \times \; 2\,sec\theta\; sec\theta\; tan\theta \right] = 0$$

$$=> 20 - 2 \times \frac{2000}{V^2} \times tan\theta + \; \lambda \left[9 - 2 \; \times \frac{405}{V^2} \times tan\theta \right] = \; 0$$

$$put\; \lambda = \; -\frac{2000}{405}$$

$$=> \; 20 - 2 \times \frac{2000}{V^2} \times tan\theta - \frac{2000}{405} \left[9 - 2 \; \times \frac{405}{V^2} \times tan\theta \right] = \; 0$$

$$=> 20 - 2 \times \frac{2000}{V^2} \times tan\theta - \frac{2000}{405} \times 9 \; + \; 2 \times \frac{2000}{V^2} \times tan\theta \; = \; 0$$

Here we see that the variable terms cancel out and we are left with $20 = \frac{1800}{405}$, which is clearly inconsistent. Since the manual approach did not yield proper results, MATLAB was used for getting the optimized values from the error equation with different inbuilt solvers.

Matlab provides different solvers for a variety of optimization problems. Fmincon can be used to find a minimum of a constrained non-linear multivariable function, internally using sequential quadratic programming algorithm. The SQP algorithm combines the objective and constraint functions into a merit function. The algorithm attempts to minimize the merit function subject to relaxed constraints.

'patternsearch' solver uses generalized pattern search (GPS) algorithm to find a sequence of points x_0, x_1 , x_2 ... that approach an optimal point.

Optimization using fmincon and patternsearch solvers used for minimizing the error where the ball gets into the goal, using 2.2m as the optimal goal entry height:

Objective Function for the optimizer:

$$function\; f = objectfun(X)$$

$$f \;=\; abs\left(\left(\left(\tan\left(X(2) * \left(\frac{\pi}{180}\right)\right) \times \; 20 - \left(\frac{9.8\; \times (20^2)}{\left(2 \times \left(X(1)^2 \; \times ((\cos\left(X(2) \times \frac{\pi}{180}\right))^2\; \right)\right)}\right)\right)\right) - 2.2\right);$$

Where, X(1) and X(2) are the variables velocity and angle.

Non-Linear Constraints for error minimization:

Constraint is defined by taking negative value of the height of the ball at the wall and adding it to the defined wall height, which should be less than or equal to 0:

$$function[c, ceq] = nonlnrconstr(X)$$

$$c = [\, - \left(\tan\left(X(2) * \left(\frac{\pi}{180}\right)\right) \times \; 9 - \left(\frac{9.8\; \times (9^2)}{\left(2 \times \left(X(1)^2 \; \times ((\cos\left(X(2) \times \frac{\pi}{180}\right))^2\; \right)\right)}\right)\right) + \; 1.9]$$

$$ceq = [\,]$$

Fmincon solver returned following optimized results with minimum error:

(Velocity: 16.894, Angle: 29.324)

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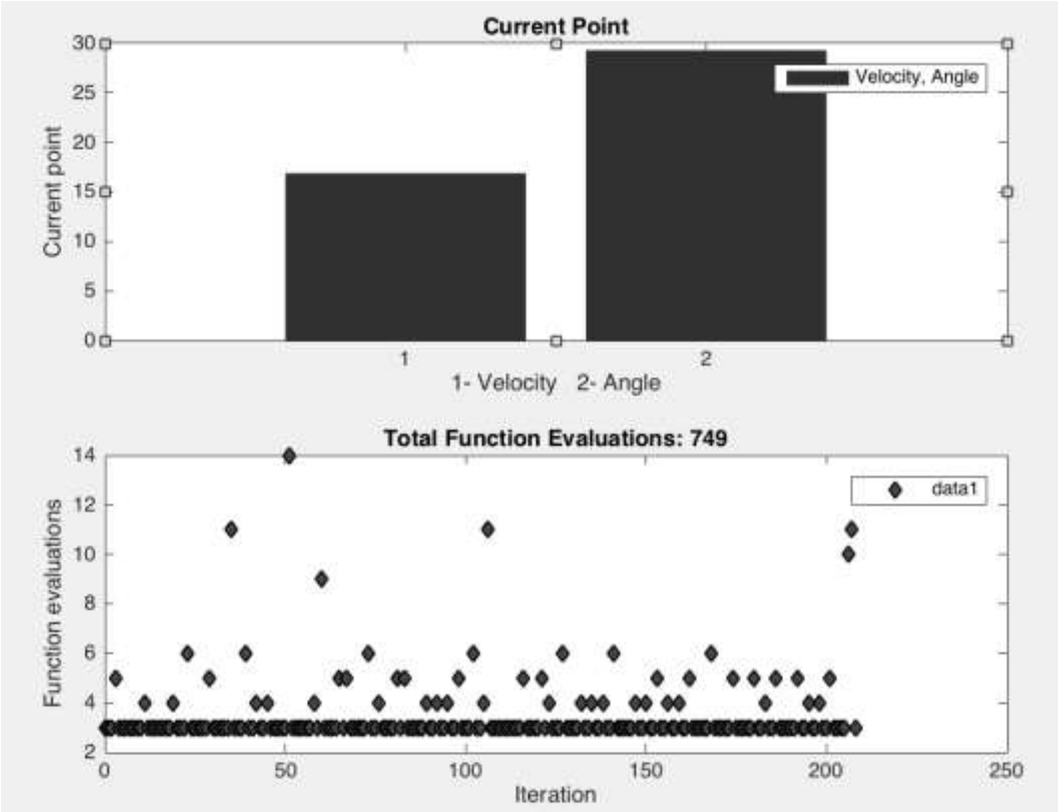

Fig.4. fmincon solver results for Error minimization

Substituting the solution derived from above solver we see the following ball positions:

$Height\ of\ the\ ball\ at\ the\ wall\ =\ 3.2414m$

$Height\ of\ the\ ball\ at\ the\ goal\ =\ 2.2002m$

$Time\ to\ reach\ the\ goal\ =\ 1.3579s$

Patternsearch: (Velocity: 22.654, Angle: 16.459)

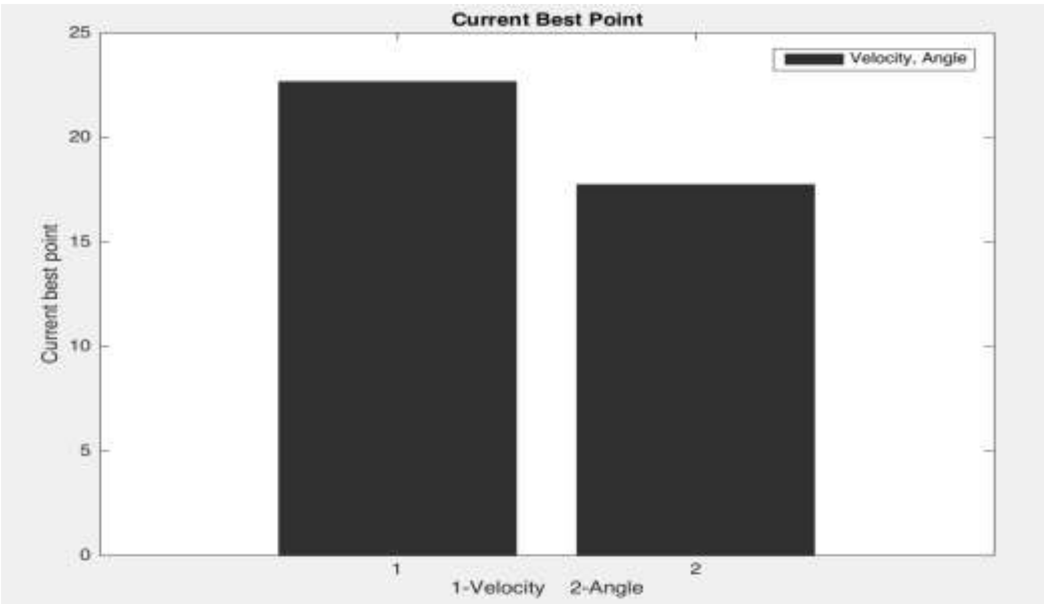

Fig.5. patternsearch solver results for Error minimization

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Substituting the solution derived from above solver we see the following ball positions:

$Height\ of\ the\ ball\ at\ the\ wall\ =\ 2.0432m$

$Height\ of\ the\ ball\ at\ the\ goal\ =\ 2.2000m$

$Time\ to\ reach\ the\ goal\ =\ 0.9261s$

Using patternsearch solver, we were able to clear the wall and also get the perfect optimal goal in less time compared to the fmincon results.

VII. TIME OPTIMIZATION

It is also important to note that the position with which the ball enters the net is not the only determining factor of whether a goal will be successful. The time taken for the ball to reach the goal also makes a difference in the goal success rate. Less time means that the goalkeeper has less time to react and therefore the chances of saving decrease. Therefore it is important to minimise time when finding the optimal initial angle and velocity. A similar process as was done when minimising error can be used here, except that an additional constraint for the error has to be applied to make sure the ball still reaches the above optimal range we set. We only want to consider those paths whose error is within 0.1m. This can be seen from the following equations.

This function minimizes on time using wall height and the goal positions as constraints and the objective function uses the projectile motion time calculation and getting the value of time based on velocity, angle and horizontal displacement.

Objective Function for the optimizer:

$function\ f = objectfun(X)$

$$f = \left(\left(X(1) \times \sin\left(X(2) \times \frac{\pi}{180}\right)\right) + \left(\sqrt{X(1)^2 \times (sin\left(X(2) \times \frac{\pi}{180}\right))^2 - (4.4 \times 9.8)}\right)\right)/9.8;$$

Where, X(1) and X(2) are the variables velocity and angle.

Non-Linear Constraints for time minimization:

In addition to the wall constraint defined in the previous optimization, we have to add another constraint to minimize the error of the ball entry at the goal by giving it a margin of 0.1m deviation from the 2.2m ideal height for the ball to enter the goal:

$function[c, ceq] = nonlnrconstr(X)$

$c = [$

$$-\left(\tan\left(X(2) * \left(\frac{\pi}{180}\right)\right) \times 9 - \left(\frac{9.8 \times (9^2)}{\left(2 \times \left(X(1)^2 \times ((\cos\left(X(2) \times \frac{\pi}{180}\right))^2\right)\right)}\right)\right) + 1.9;$$

$$abs\left(-\left(\tan\left(X(2) * \left(\frac{\pi}{180}\right)\right) \times 20 - \left(\frac{9.8 \times (20^2)}{\left(2 \times \left(X(1)^2 \times ((\cos\left(X(2) \times \frac{\pi}{180}\right))^2\right)\right)}\right)\right) + 2.2\right) - 0.1;$$

$$-\left(X(1) \times \sin\left(X(2) \times \left(\frac{\pi}{180}\right)\right)\right) + 6.57;$$

$]$

$ceq = [\,]$

Fmincon solver returned following results: (Velocity: 23.654, Angle: 16.459)

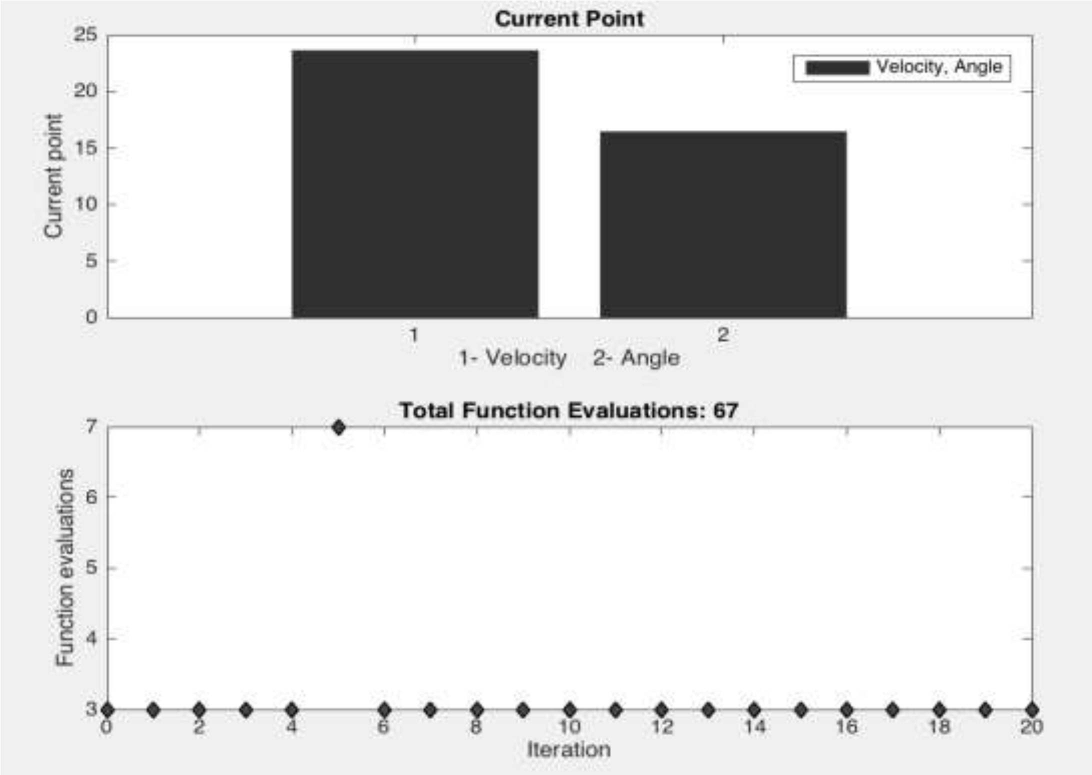

Fig.6. fmincon results for Time minimization

Substituting the solution derived from above solver we see the following ball positions:

$Height\ of\ the\ ball\ at\ the\ wall\ =\ 1.8999m$

$Height\ of\ the\ ball\ at\ the\ goal\ =\ 2.0999m$

$Total\ time\ to\ reach\ the\ goal\ =\ 0.8206s$

patternsearch solver returned following results: (Velocity: 23.843, Angle: 16.384)

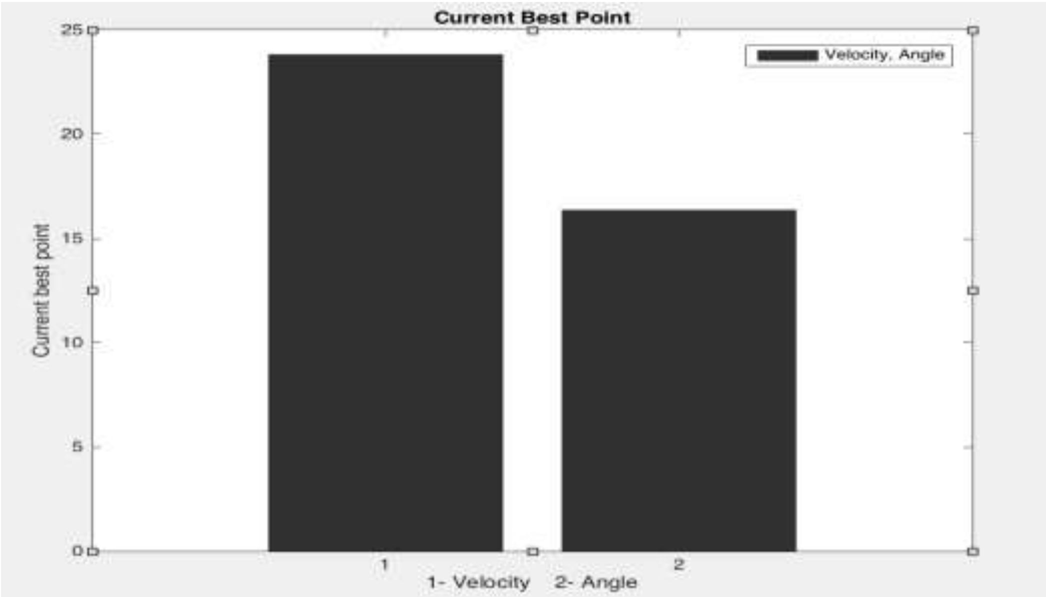

Fig.7. patternsearch results for Time minimization

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Substituting the solution derived from above solver we see the following ball positions:

$$Height\ of\ the\ ball\ at\ the\ wall\ =\ 1.9m$$

$$Height\ of\ the\ ball\ at\ the\ goal\ =\ 2.1345$$

$$Time\ to\ reach\ the\ goal\ =\ 0.8346s$$

Both the solvers returned approximately same results of angle and velocity with respect to time while meeting the constraints reasonably.

VIII. EMPIRICAL APPROACH

Real field data has been captured from empirical investigation done in the field. A sample of Free Kicks was taken with different initial velocity and angles. This was mapped using Tracker software and Adidas Snapshot. The table below has mapped the goal success rate at intervals of initial velocity and angle:

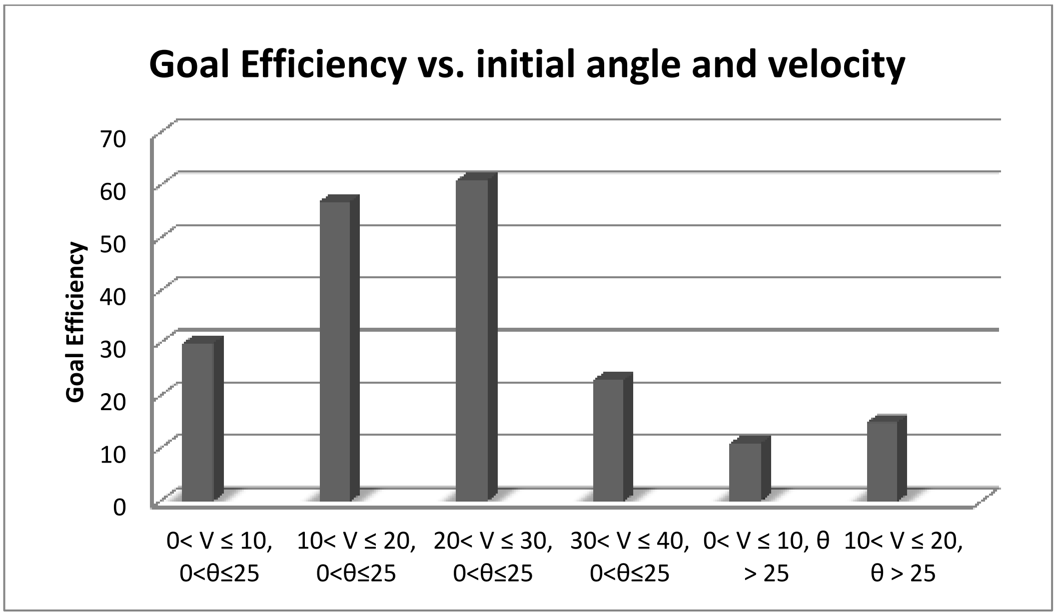

Fig.8. Goal Efficiency vs. initial angle and velocity

There is difference in the values obtained theoretically Vs. empirically, because there are several factors that have not been taken into account such as the spin of the ball, wind factors and technique of kicking. However, we can see from the general trend that goal efficiency is maximised around the range of:

$$10\ m/s\ <\ V_o\ <30\ m/s\ and$$

$$0^o\ <\ \theta <\ 25^o$$

This generally coincides with the data from the earlier, theoretical approach taken.

Fig. 9. Field Data Collection

IX. CONCLUSION

From the results of the various methods of solutions, one can note the nuances in the results data values. Performed empirically, a slight amount of deviation from expected theoretical results is noted, and this could result for several reasons. The spin of the ball, along with the wind factors has not been considered and thus could be a future improvement to the project. Furthermore, varying techniques of the kickers may also influence the trajectory of the ball, despite having the same initial angle and trajectory. Thus, taking a bigger sample of empirical data with several kickers will improve the results.

At a time when soccer players around the world are trying to 'perfect' their game, Mathematics offers the means for significant enhancement of Free Kick performance for all types of players. Integrated within the training system, such mathematical results could truly revolutionise the game and thus the notion of optimizing a Free Kick should be seriously considered.

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